RADIATIONAL INVERSIONS AND SURFACE TEMPERATURE CHANGES

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ABSTRACT

When air stagnates in dark polar regions, an inversion forms. The change of the inversion magnitude is studied under the assumption that the snow surface radiates about as much energy as it receives from the atmosphere. It turns out that the inversion magnitude may either decrease or increase as the surface temperature falls, depending on the rate of change of atmospheric "emissivity" with air temperature.

1. INTRODUCTION

Wexler [1, 2] has studied the formation of inversions during the long polar night. An examination of his papers shows that under some conditions the inversion magnitude, ΔT, decreases, and for other conditions ΔT increases as the surface temperature, T_s decreases. The inversion magnitude is given by

$$\Delta T = T_a - T_s \tag{1}$$

where T_a is the temperature of an isothermal layer of air which lies above a colder surface.

Actually, Wexler studied the following phenomenon. He assumed that during the long polar night a relatively unstable maritime airmass stagnated over a snow surface with initial surface temperature 273° K. He then tried to compute the future course of the air temperature, T_a, and surface temperature, T_s, assuming that only radiation was acting. The model he adopted stipulated that T_s falls very quickly to a quasi-equilibrium value determined by the downcoming radiation from the air. Thereafter, T_s decreases slowly. The air temperature then also falls slowly, forming an isothermal layer with temperature, T_a, above the ground; above the isothermal layer, the original maritime air temperature prevails. In his 1936 paper, with the aid of water vapor absorption coefficients measured by Weber and Randall [3], Wexler [1] found that ΔT decreased as T_s decreased, after T_s had reached the quasi-equilibrium value. Later, in 1941, Wexler [2] found that if he used the newly developed Elsasser diagram for saturated air ΔT increased as T_s decreased. Whether or not ΔT increases or decreases depends critically on the *rate of change* of the atmospheric radiation with air temperature.

It is the purpose of this paper to inquire under what conditions ΔT would *increase* with decreasing T_s, and under what conditions ΔT would *decrease* with decreasing T_s, in an individual mass of air cooling by radiation only.

2. EMISSIVITY AND RADIATION

From the assumed state of quasi-radiative equilibrium between the snow surface and the overlying air, we get

$$R = \sigma T_s^4 = k\sigma T_a^4 \tag{2}$$

$$T_s = k^{1/4} T_a \tag{3}$$

where k is an overall "emissivity" of the atmosphere, and we assume that the main part of the downward atmospheric radiation comes form the isothermal layer; σ is the Stefan-Boltzmann constant.

Since $\Delta T = T_a - T_s$, by substituting for T_s from equation (3), we get

$$\Delta T = T_a(1 - k^{1/4}) \tag{4}$$

The problem now is to find under what conditions of radiation, R, and therefore of atmospheric emissivity, k, ΔT increases with decreasing T_s, and when ΔT decreases with decreasing T_s. We can get the limiting condition by making ΔT constant.

Thus equation (4) becomes

$$T_a = \frac{\Delta_c T}{(1 - k^{1/4})} \tag{5}$$

where Δ_c denotes that ΔT is constant with variations in T_s or T_a.* Equation (5) shows the relation between the "emissivity", k, and isothermal air temperature, T_a, which is required in order for the inversion magnitude, ΔT, to remain constant as the surface temperature, T_s, drops. This relation of k vs. T_a is shown in figure 1 for a range of k and T_a which is found in Wexler's studies. Figure 1 shows that the emissivity of the isothermal air may decrease with decreasing air temperature even though the

*Under the conditions that ΔT=constant with T_0,

$$\frac{\partial \Delta T}{\partial T_s} = \frac{\partial \Delta T}{\partial T_a} = 0$$

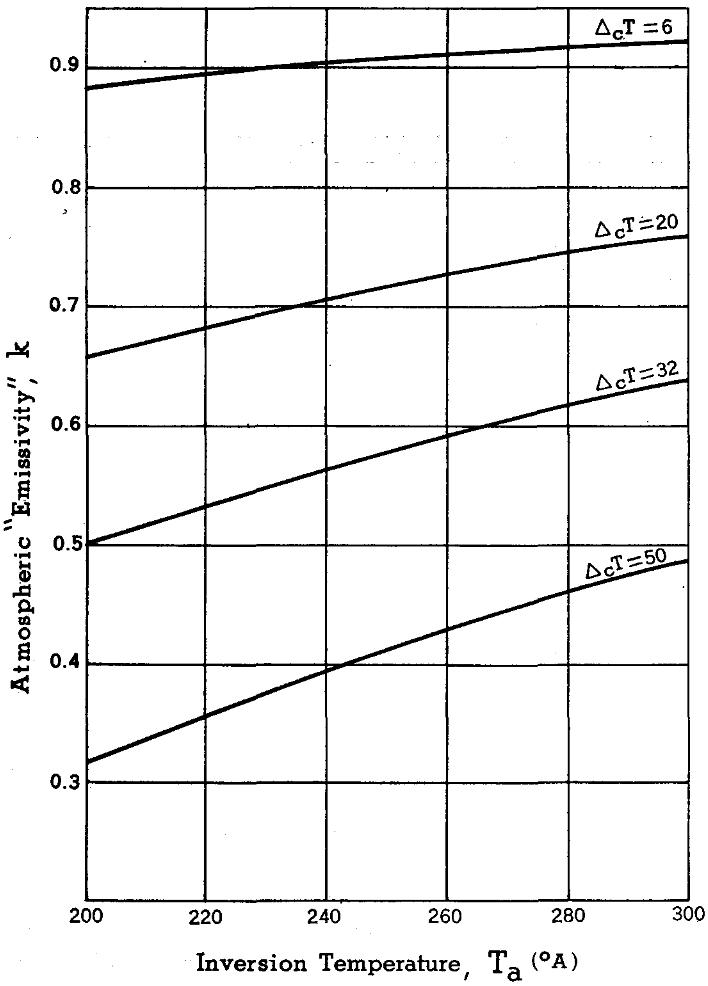

FIGURE 1.—Variations of atmospheric "emissivity," k, with inversion temperature, T_a, when the inversion magnitude, $\Delta_c T$, does not change with surface temperature, T_s.

inversion magnitude is not changing as surface temperature falls. However, if the slope of the k vs. T_a lines is steeper than those in figure 1 (that is, if k decreases faster with decreasing T_a), ΔT will increase as T_s decreases; if the slope is less steep, ΔT will decrease.

The conditions on the atmospheric radiation R, for ΔT=constant, can be readily found from equation (2). As background, however, we should look at R vs. T curves from Wexler's 1941 paper [2]. In figure 2, curves a, c, and d are taken from that paper. Curve a represents the black body radiation of a snow surface, plotted against T_s; curves c and d represent the downward atmospheric radiation computed in various ways and plotted against T_a. In curve c, T_a represents "the highest mean temperature of any layer of air containing 1 mm. of precipitable water and normal CO_2 content." In the computation of R for curve c the absorption coefficients of Weber and Randall [3] were used. Curve d was ". . . computed by use of the Elsasser diagram in the following way: the radiation coming from a saturated atmosphere in convective equilibrium with an ocean surface of 0° C. is designated by the ordinate of the extreme right hand point of curve (d), while the ordinates of other points of this curve refer to the radiation coming from atmospheres whose lower portions are isothermal at temperatures corresponding to the abscissae of the curve and whose upper portions follow the original convective equilibrium curve." The values of T_a and T_s, tabulated by Wexler [2] show that

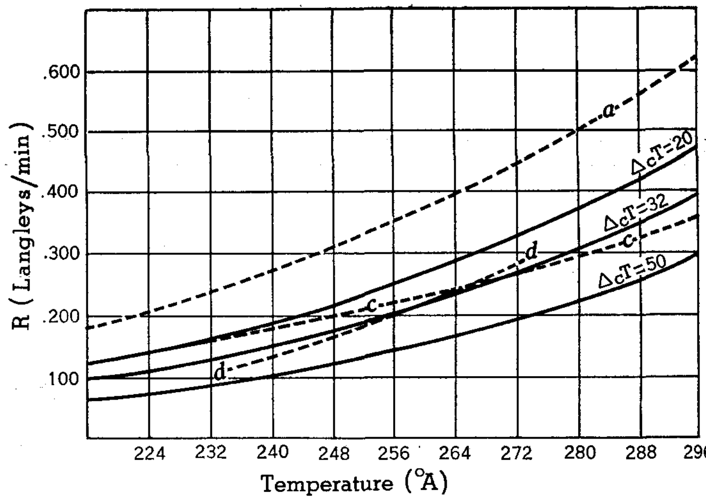

FIGURE 2.—Radiation, R, vs. temperature. Curve "a" refers to radiation of a black body, in langley/min., against surface temperature, T_s. Curves "c" and "d" refer to radiation from the atmosphere related to air temperature (see text). (Curves a, c, d from [2].) Curves labeled $\Delta_c T$ refer to radition from air when the inversion magnitude does not change with surface temperature.

$\frac{\partial \Delta T}{\partial T_s} > 0$ for curve c and less than zero for curve d.**

This result is to be expected. From the definition of $\Delta_c T$, by substituting for T_s in equation (2), we get

$$R = \sigma (T_a - \Delta_c T)^4$$

The values of R vs. T_a for several values of $\Delta_c T$ are plotted in figure 2. These curves, of course, apply only when ΔT is constant with changes in T_a or T_s. If any lines in figure (2) converge toward curve a, as T_a decreases, more rapidly than the $\Delta_c T$ curves, then $\frac{\partial \Delta T}{\partial T_s} > 0$; if such lines diverge from curve a more rapidly than the $\Delta_c T$ curves then $\frac{\partial \Delta T}{\partial T_s} < 0$. Thus, curves c and d represent the type of variation of $\frac{\partial \Delta T}{\partial T_s}$ to be expected; that is, for curve c the inversion decreases with decreasing surface temperature; the opposite is true for curve d. It is easy to verify from figure 2, that in curve c, k is very nearly constant. But for curve d, k decreases more rapidly with decreasing T_a than indicated by the curves in figure 1.

**Wexler's table shows this for $T_s < -40°$ C. However his curve d shows it for all values of T_s.

Thus, we might expect in our simplified model, the condition for $\frac{\partial \Delta T}{\partial T_s} < 0$ is that the "emissivity" of the atmosphere should decrease more rapidily with decreasing air temperature than the curves in figure 1.

3. DISCUSSION

Although curves c and d represent only computation with special assumptions and conditions, it is not hard to visualize that in actual atmospheres the emissivity may vary with T_a in the same way as implied in curve c or in curve d. For example, evaporation and condensation may do this. Therefore, it is not valid to generalize that the inversion magnitude either increases or decreases with decreasing surface temperature after quasi-equilibrium has been reached. What happens in any particular case depends on the radiation properties of the atmosphere and how they change with temperature.

Moreover comparison of T_s with ΔT with the aid of many different radio-soundings for many different meteorological conditions and airmasses, does not shed any light on the direction of change of ΔT with T_s in a single airmass. Nevertheless Belmont [4] has concluded from his own findings that his results conflict with Wexler's theoretical results. Belmont compiled measurements of inversions and found that, on the average, the magnitude of "radiation" inversions increases as T_s decreases, especially for low values of T_s. Since radiosonde data were used, the results do not refer to the history of a single mass of air during the time when the air and surface cool. The results compare, instead, the temperatures of many different masses of air with the simultaneous surface temperatures. Therefore, Belmont's results are, in any case, not comparable with Wexler's results since each was dealing with a different problem. But it should nevertheless be pointed out that Wexler's results include *both* the increase *and* decrease of ΔT with decreasing T_s, depending on the rate of change of atmospheric emissivity with air temperature.

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